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6. AUTHOR(S)
Vladimir A. Fonoberov
Maria Fonoberova
Igor Mezic
Jadranka Mezic

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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
AIMdyn, Inc.
1919 State St., Ste. 207, Santa Barbara, CA 93101, USA

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AFOSR
875 N. Randolph St.
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14. ABSTRACT
An agent-based model of civil violence/criminal activity was developed. The agent based model was found to be in good agreement with FBI data on crime and violence in 5660 U.S. cities. In particular, the proportion of law enforcement officers required to maintain a steady low level of criminal activity was found to increase with the size of the population. Reducing the number of law enforcement officers below a critical level can rapidly increase the incidence of violent/criminal activity. Application of global sensitivity analysis to the agent-based model identified the law enforcement officer vision as the most significant parameter. A method to speed-up the model evaluation was developed by employing stochastic differential equations. Jailed citizens and active citizens served as two coarse variables in the reduced model. The effect of preferential gathering sites attracting active citizens was studied. It was found that increased density of citizens around the gathering cites leads to increased violence/criminal activity in the city. The effect of citizen's remote vision via a small world network was also studied. It was found that local vision is more important in creating outbursts of activity than the possibility of being connected to long-distance neighbors.

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FINAL REPORT FOR AFOSR GRANT FA9550-08-1-0217

Maria Fonoberova[*], *Vladimir A. Fonoberov*[*] *(PI), Igor Mezić*[†], *Jadranka Mezić*[*]

[*] Aimdyn, Inc., 1919 State St., Ste. 207, Santa Barbara, CA 93101, USA
[†] Center for Control, Dynamical Systems and Computation, University of California - Santa Barbara, Santa Barbara, CA 93106, USA

Abstract

An agent-based model of civil violence / criminal activity was proposed and validated. The analysis of data on crime and violence for 5,660 U.S. cities over the period of 2005-2009 was performed and the following trends were uncovered. The proportion of law enforcement officers required to maintain a steady low level of criminal activity increases with the size of the population of the city. The number of criminal/violent events per 1,000 inhabitants of a city shows non-monotonic behavior with size of the population. The strong dependence on size of the population was found, which leads to partially irrational behavior on the part of citizens. The nature of violence changes from global outbursts of criminal/violent activity in small cities to spatio-temporally distributed, decentralized outbursts of activity in large cities, indicating that in order to maintain peace, bigger cities need larger ratio of law enforcement officers than smaller cities. Tipping points for communities of all sizes in the model were observed: reducing the number of law enforcement officers below a critical level can rapidly increase the incidence of criminal/violent activity, what is in agreement with the data.

We employed global sensitivity analysis as a tool for evaluating and validating agent-based models. The general scheme for applying the global sensitivity analysis to agent-based models was proposed on an example of a civil violence/criminal activity agent-based model. The "vision" of law enforcement officers was identified as the most significant parameter in the model. Other important parameters include density of law enforcement officers, citizen "vision", and speed of law enforcement officers. Reduced-order models for agent-based models were developed. The dependence of rate of violence and outburst waiting time on the model parameters was investigated.

In another model reduction approach, the civil violence model was reduced to two-variable dynamics characterized by a two-dimensional stochastic differential equation. The populations of jailed citizens and active citizens served as the two coarse variables in the reduced model. A detailed relationship between the two coarse variables and other model variables was revealed, based on which a procedure was provided to generate the whole system scenario merely from values of the two variables. The reduced model can replicate the punctuated equilibrium behavior emerging in the original model and can also save simulation time by approximately 95%.

In a modification of our agent-based model, preferential gathering sites were introduced on the lattice so that active citizens move in their Moore neighborhood in the direction of the closest preferential gathering site. This change incorporates the effects of aggregation into the model. The probability with which active citizens move to the preferential gathering site was also introduced. The bigger the probability, the denser clusters of active citizens are formed around the sites.

Finally, a small world network was introduced to the agent-based model of civil violence / criminal activity. Approximately 10% of citizens that would be in the citizen vision were instead connected thorough the small world network. In the test case of small numbers of law enforcement officers per 1,000 citizens the peak number of active citizens and the number of violent outbursts per year increase substantially after introducing the small world network. In the case of realistic numbers of law enforcement officers per 1,000 citizens, the number of active citizens per day and the number of violent outbursts per year turned out to be much lower for the case with small world network (and smaller local citizen vision), suggesting that local vision is more important in creating outbursts of activity than the possibility of being connected to long-distance neighbors.

Contents

1 Summary of the Agent-Based Model of Civil Violence / Criminal Activity

There are two kinds of agents in the model: citizens and law enforcement officers (LEO's). The agent-based model described in Ref. [1] was used as a starting point in developing the model described below. Citizens are civilian members of the population who bear no responsibility for controlling crime. By contrast, LEOs are forces of the authority who's primary function is to detect criminal activity and attempt to curtail it by incapacitating individuals committing crimes. Citizens can become active in criminal and/or violent activity, or stay quiescent depending on a number of factors described below. LEOs always seek out and arrest active citizens. All events transpire on a lattice with periodic boundaries. All citizens move once per day, and LEO's move several times per day. Figure 1 gives a schematic representation of the lattice.

[Figure 1 here.]

Each citizen is assigned the hardship H drawn from the uniform distribution $U(0, 1)$. The hardship is heterogeneous across citizens and is constant for each citizen. The perceived legitimacy L of the law enforcement authority is equal across citizens and can be between 0 and 1. The hardship and the legitimacy are used to define citizen's grievance $E = H(1 - L)$. Some citizens are more willing to pursue criminal activity than others, this is encoded by the risk aversion K. Each individual's risk aversion is drawn from $U(0, 1)$ and is constant for each citizen. The citizen's vision v is a circle of radius v that comprises lattice positions that the citizen is able to inspect. It is equal across citizens. The citizen's perceived net risk N is defined as follows: $N = KP$, where P is a function of the ratio of LEO's to active citizens. If for a law-abiding citizen the difference $E - N$ exceeds T, where T is some threshold, then the citizen becomes criminally active. If, for an active citizen, the difference $E - N$ exceeds T, then the citizen stays active. Otherwise, he/she becomes law-abiding. In summary, the citizen's rule for being active or quiescent is the following: If $E - N > T$ be active; otherwise, be quiescent.
The citizen's perceived risk function P is defined as:

$$P(C/A) = 1 - exp(-k'(C/A)) \sum_{i=0}^{15} \frac{(k'(C/A))^i}{i!}, \tag{1}$$

where A is the number of active citizens (including self) within citizen's vision, C is the number of LEO's within citizen's vision, constant $k' = 62.6716$ is found from the condition that $P(1/4) = 0.5$ (see Figure 2).

[Figure 2 here.]

The perceived risk is in fact zero up to a threshold value, after which it increases monotonically, thus giving it a sigmoidal shape. The sigmoidal shape encodes a level of irrational behavior by citizens, where the real risk of being incarcerated is being diminished by the proportion of others in the same situation. The law enforcement operates through the rule that a LEO agent arrests the nearest active citizen. This rule leads to dynamics in which local crime hotspots attract police action, thus reflecting the modern problem-oriented policing law enforcement strategies. For more information on roots of the model in criminology theory see [2].
In general, citizens are less likely to engage in violence as the local ratio between number of LEO's and active citizens increases due to a fear of being identified and incarcerated. The citizen state (active or quiescent) can be regarded as a function of threshold T. Figure 3 shows the structure of population depending on threshold. In particular, in the case when $T > 1 - L$, all citizens are always quiescent regardless of the lattice situation; we call them "never active" and denote their number as G. In the case when $T < -1$, all citizens are "always active", we denote their number as R. When $-1 < T < 0$, $G = 0$ and all citizens are either "always active" or "conditionally active" (active or quiescent depending on the lattice situation). The case when $0 < T < 1 - L$, is the most realistic one with all three groups of population present. In practice the threshold T and legitimacy L for the model run can be found using statistical data on fractions of R and G in the population:

$$T = \frac{2R}{1 - G}; \tag{2}$$

$$L = 1 - \frac{2R}{G(1-G)}. \tag{3}$$

[Figure 3 here.]

LEO's seek out and arrest active citizens. The LEO's vision w is a circle of radius w that comprises lattice positions that the LEO is able to inspect. It is equal across LEO's. The LEO's rule is the following: Inspect all sites within w and arrest the nearest active citizen. Jail terms for arrested actives are assigned randomly from $U(0, J_{max})$, where J_{max} is the maximum jail term. Citizens and LEO's move on the lattice by using the following movement rule: Pick a random neighboring location on the lattice (from Moore neighborhood), if that location is unoccupied - move there, if the location is occupied - stay put.
In such a way, citizens can be active in criminal and/or violent activity, stay quiescent, or be jailed (see Figure 4).

[Figure 4 here.]

The procedure of a run is the following. A citizen or a LEO is selected at random. The probability of selecting a LEO is higher according to the number of moves a LEO can make per day. If the selected person is a non-jailed citizen, then he/she moves according to the movement rule; if the citizen is in jail, then the days at jail are calculated and if the number of days at jail is equal to the assigned jail term the released citizen is put on a random unoccupied site on the lattice. After that the state of the citizen is calculated depending on the current lattice situation. If the selected person is a LEO, then he/she inspects all sites within the vision, arrests the nearest active citizen (if any) and jumps to the location of the arrested by him citizen. The jailed citizens are placed outside the lattice. Then the LEO moves under the movement rule. The model iterates this procedure until the simulation time is reached.

2 Results Reported During Years 1-2, Submitted for Publication

2.1 Nonlinear Dynamics of Crime and Violence in Urban Settings

In [2] we performed analysis of data on crime and violence for 5,660 U.S. cities over the period of 2005-2009 and uncovered the following trends: 1) The proportion of law enforcement officers required to maintain a steady low level of criminal activity increases with the size of the population of the city; 2) The number of criminal/violent events per 1,000 inhabitants of a city shows non-monotonic behavior with size of the population. We find strong dependence on size of the population, which leads to partially irrational behavior on the part of citizens. The nature of violence changes from global outbursts of criminal/violent activity in small cities to spatio-temporally distributed, decentralized outbursts of activity in large cities, indicating that in order to maintain peace, bigger cities need larger ratio of law enforcement officers than smaller cities. We also observed existence of tipping points for communities of all sizes in the model: reducing the number of law enforcement officers below a critical level can rapidly increase the incidence of criminal/violent activity. However surprising, these trends are in agreement with the data.

2.2 Global Sensitivity/Uncertainty Analysis for Agent-Based Models

In [3] we considered the sensitivity analysis as a tool for evaluating and validating agent-based models. We presented the general scheme for applying the global sensitivity analysis to agent-based models on an example of a civil violence/criminal activity agent-based model. We identified law enforcement officer vision as the most significant parameter in the model and uncover contributions to outputs of other important parameters such as LEO density, citizen vision, and LEO speed. We discussed development of reduced-order models for agent-based models. We showed the model reduction process and investigated rate of violence and outbursts waiting time dependence on the model parameters.

2.3 A Case Study of Model Reduction for Agent-Based Social Simulation: Coarse-Graining a Civil Violence Model

In [4] the civil violence model was reduced to a two-variable dynamics characterized by a two-dimensional stochastic differential equation. The populations of jailed citizens and active citizens serve as the two coarse variables in the reduced model. A detailed relationship between the two coarse variables and other model variables is revealed, based on which a procedure is provided to generate the whole system scenario merely from values of the two variables. The reduced model can replicate the punctuated equilibrium behavior emerging in the original model and can also save simulation time by approximately 95%.

3 Effect of Preferential Gathering Sites Attracting Active Citizens

In Sections 3-4 we describe results obtained during Year 3 of the project. As a small update to the model, we changed the rules by which the jailed citizen is released from jail. The citizen is now released from jail to the same spot where he/she was arrested or, if that spot is occupied, to the spot closest to the spot where the citizen was arrested. This model update is needed to accurately study the effects of preferential gathering sites and remote vision via a small-world network.

In this section we introduce preferential gathering sites on the lattice so that active citizens move in their Moore neighborhood in the direction of the closest preferential gathering site. This change incorporates into the model the effects of aggregation, important to capture possibility of strong but localized outbursts in certain areas.

We consider the following model outcomes: number of active citizens per day in percent, number of revolutions per 1,000 days, peak number of active citizens in percent, number of violent outbursts per year, rate of violence. In the following we introduce a threshold (e.g. 5% of population) and call an outburst large-scale if it exceeds the threshold. The number of active citizens per day is the total number of citizens active at the end of the day. The number of revolutions per 1,000 days is the total number of large-scale outbursts per 1,000 days. The peak number of active citizens is the maximum number of active citizens during the outburst. The number of violent outbursts per year is inversely proportional to the waiting time between two large-scale outbursts. The rate of violence is the number of citizens active at the end of the day or jailed during the day per 1,000 citizens. The exact definitions of the outputs averaged over all realizations are given in the following table. These outputs are calculated and plotted, depending on the situation, for specific time intervals.

Nr.	Output Name	Output Definition
1	Number of active citizens per day in percent	Sum of total number of citizens active at the end of the day for the specified time period / population size * 100 / time period
2	Number of revolutions per 1,000 days	Total number of revolutions per 1,000 days
3	Peak number of active citizens	Averaged maximum number of active citizens during the outburst over all outbursts / population size * 100
4	Number of violent outbursts per year	365 / Averaged waiting time between two outbursts over all outbursts
5	Rate of violence	Sum of total number of citizens active at the end of the day or jailed during the day for the specified time period / population size * 1000 / time period

3.1 Test Case with Small Number of LEOs per 1,000 Citizens

We consider that citizens density is 0.7 (i.e. citizens occupy 70% of the lattice); number of LEOs per 1,000 citizens is 1.27; citizen vision is 14; LEO vision is 14; LEO speed is 4 (i.e. LEOs move 4 times per day); maximum jail term is 120 days; fraction of always active citizens R = 0.025; fraction of never active citizens G = 0.45; fraction of conditionally active citizens is 0.525. Threshold T and legitimacy L can be calculated by using formulas (2) and (3). All always active citizens are initially in jail. We considered six lattices sizes (100x100, 200x200, 300x300, 400x400, 500x500, 600x600) with the corresponding real population of 63,000; 252,000; 567,000; 1,008,000; 1,575,000; 2,268,000. We assume that one citizen agent in our model represents 9 real citizens. 10,101 days on the lattice were computed for each simulation. Each experiment was repeated at least 16 times. For testing purposes the number of LEOs per 1,000 citizens is selected in such a way so that the rate of violence is high.

We compared cases with no preferential gathering sites and 1, 2, 3, 4, 5, 10, 20, and 30 sites. In the case of no sites, the large-scale outbursts of activity are relatively rare for big lattices (see Figure 5). With the introducing of small number of sites, the large-scale outbursts of activity happen much more often after some amount of time (see Figure 6), which can be explained by the fact that with time active and conditionally active citizens gather around sites in such a way forming clusters of activity (see Figure 7). After the larger number of sites is introduced, the large-scale outbursts of activity happen rarely, because active citizens are spread on the lattice on the areas surrounding the sites and don't see each other (see Figure 8).

[Figure 5 here.]

[Figure 6 here.]

[Figure 7 here.]

[Figure 8 here.]

Number of active citizens per day at first increases with the number of preferential gathering sites introduced but after some amount of time, depending on the lattice size, it stabilizes for different numbers of introduced sites (see Figure 9).

[Figure 9 here.]

Number of revolutions per 1,000 days increases with the number of introduced preferential gathering sites for the cases of 1 to 5 sites, what can be explained by the fact that citizens form clusters of activity around introduced sites and in such a way there is a much bigger possibility for them to become active. When a larger number of sites is introduced, the number of revolutions decreases, because the citizens are more spread on the lattice and the possibility of forming big clusters of activity is smaller (see Figure 10).

[Figure 10 here.]

Peak number of active citizens does not differ significantly for the cases with and without preferential gathering sites. The bigger lattice is, the closer peak numbers of actives are. In general the peak number of active citizens is around 5% for big lattices, as there are no too many large-scale outbursts in these cases (see Figure 11).

[Figure 11 here.]

Number of violent outbursts per year increases with the number of introduced preferential gathering sites for the cases of 1 to 5 sites. This means that the more preferential gathering sites are introduced, the smaller is the waiting time between revolutions. For the larger number of introduced preferential gathering sites the number of violent outbursts per year decreases, what means that the waiting time between large-scale outbursts increases (see Figure 12).

[Figure 12 here.]

Rate of violence is smaller without preferential gathering sites and does not differ significantly for different numbers of preferential gathering sites. At first the rate of violence slightly increases with the number of preferential gathering sites introduced but after some amount of time, depending on the lattice size, it stabilizes for different numbers of sites introduced in the same way as for the number of active citizens per day (see Figure 13).

[Figure 13 here.]

In the following we plot the dependence of all model outcomes on the lattice size during time [9000;10101] for different numbers of sites. The number of active citizens per day does not change essentially with the increase of lattice size (see Figure 14). The number of revolutions per 1,000 days peaks for lattices of average size, what can be explained by the fact that for smaller lattice sizes there are no too many outbursts of activity while for bigger lattices there are a lot of activity on the lattice but there are no above 5% outbursts of activity (see Figure 15). The peak number of active citizens is the highest for small lattices and decreases up to 5%, what is considered the limit for considering an outburst a large-scale outbursts, with the increase of lattice size (see Figure 15). The number of violent outbursts per year is the biggest for lattices of medium size (see Figure 16). The rate of violence does not differ significantly for all lattice sizes (see Figure 16).

[Figure 14 here.]

[Figure 15 here.]

[Figure 16 here.]

Above we considered that active citizens move to the preferential gathering site with 100% probability, i.e. active citizens always try to move in the direction of the preferential gathering site. In the following we introduce the probability with which active citizens try to move to the preferential gathering site. We consider probabilities of 0%, 25%, 50%, 75% and 100% for 1, 5 and 30 preferential gathering sites. As above we consider that citizens density is 0.7; number of LEOs per 1,000 citizens is 1.27; citizen vision is 14; LEO vision is 14; LEO speed is 4; maximum jail term is 120 days; fraction of always active citizens R = 0.025; fraction of never active citizens G = 0.45; fraction of conditionally active citizens is 0.525. All always active citizens are initially in jail. We considered five lattices sizes (100x100, 200x200, 300x300, 400x400, 500x500). 10,101 days on the lattice were computed for each simulation. Each experiment was repeated at least 16 times. As we can see from simulations and based on the theoretical judgement the bigger the probability is, the denser clusters of active citizens are formed around sites. As above, we consider the following model outcomes: number of active citizens per day in percent, number of revolutions per 1,000 days, peak number of active citizens in percent, number of violent outbursts per year, rate of violence. We plot all model outcomes as 2D plots in dependence on lattice size and probability during time [9000;10101].
For different numbers of preferential gathering sites and all lattice sizes above 100x100 the number of active citizens per day increases with the increase in the probability value with which active citizens move to preferential gathering sites. For different numbers of preferential gathering sites and all probabilities above 25% the number of active citizens per day at first increases with the increase in the lattice size and then gradually decreases. For example see Figure 17 for the case of five preferential gathering sites.

[Figure 17 here.]

For different numbers of preferential gathering sites the number of revolutions per 1,000 days for smaller lattices is relatively unchangeable for non-zero probabilities. For bigger lattices the number of revolutions increases with the increase of probability, although this increase is the biggest for the case of five preferential gathering sites. With the increase of the number of preferential gathering sites the clusters of activity become spread around the lattice and this case resembles the case with no sites. For different numbers of preferential gathering sites the number of revolutions per 1,000 days for all probabilities at first increases and then decreases for bigger lattices. The increase for smaller lattices and probabilities above 50% and the decrease for bigger lattices for probabilities below 50% are much more substantial in the case of five preferential gathering sites. For example see Figure 18 for the case of five preferential gathering sites. For different numbers of preferential gathering sites the peak number of active citizens is relatively stable for all lattice sizes for different probabilities and it significantly decreases for all probabilities with the increase of lattice size up to 5%, which is considered the threshold for considering the outburst as a large-scale outburst. For example see Figure 18 for the case of five preferential gathering sites.

[Figure 18 here.]

For different numbers of preferential gathering sites the number of violent outbursts per year behaves in the similar way as the number of revolutions per 1,000 days. For example see Figure 19 for the case of five preferential gathering sites. For different numbers of preferential gathering sites the rate of violence for different numbers of preferential gathering sites and all lattice sizes above 100x100 slightly increases with the increase of probability. For all probabilities the rate of violence at first increases for small lattices and then slightly decreases for bigger lattices. For example see Figure 19 for the case of five preferential gathering sites.

[Figure 19 here.]

3.2 Case with Recommended Number of LEOs per 1,000 Citizens

In the following we investigate the issue of finding appropriate number of LEOs per 1,000 citizens so that the number of actives per day is 0.2% for different probabilities and numbers of sites. By running simulations for lattice of size 200x200 and different probabilities of active citizens moving to preferential gathering sites we found that the appropriate range for number of LEOs per 1,000 citizens is from 1.1 to 2.7. As above we consider that citizens density is 0.7; citizen vision is 14; LEO vision is 14; LEO speed is 4; maximum jail term is 120 days; fraction of always active citizens R = 0.025; fraction of never active citizens G = 0.45; fraction of conditionally active citizens is 0.525. All always active citizens are initially in jail. We consider lattices of sizes 100x100, 200x200, 300x300 and probabilities of 0%, 25%, 50%, 75%, 100% of active citizens moving towards preferential gathering sites. We consider the case of five preferential gathering sites. We ran at least 16 simulations for each value of probability and each lattice size. The appropriate number of LEOs per 1,000 citizens for five preferential gathering sites are given in the following table (see Figure 20).

[Figure 20 here.]

	100x100	200x200	300x300
0%	1.623	1.807	2.049
25%	1.588	1.920	2.114
50%	1.611	1.900	2.075
75%	1.629	1.955	2.138
100%	1.642	1.979	2.174

In the following the corresponding optimal number of LEOs per 1,000 citizens is used in simulations. Simulation with the found number of LEOs per 1,000 citizens for each lattice size and probability was run at least 16 times for the duration of 10100 days discarding first 5000 days. As above we considered the following model outcomes: number of active citizens per day in percent, number of revolutions per 1,000 days, peak number of active citizens in percent, number of violent outbursts per year, rate of violence. We plot all model outcomes in dependence on lattice size and probability during time [9000;10101] (see Figures 21 - 25).

[Figure 21 here.]

[Figure 22 here.]

[Figure 23 here.]

[Figure 24 here.]

[Figure 25 here.]

The noise in the figures is due to a relatively small number of realizations (16). The values of the number of active citizens per day, the rate of violence, the number of revolutions per 1,000 days and the number of violent outbursts per year for all lattice sizes are the smallest for the largest probability values with which active citizens move to preferential gathering sites and peak for the medium probability values. The values of the number of active citizens per day, the rate of violence, the number of revolutions per 1,000 days and the number of violent outbursts per year for all probabilities above 25% at first increases with the increase in the lattice size and then gradually decreases. The peak number of actives for all lattice sizes is the smallest for large values of probabilities. The peak number of actives for all probabilities with which active citizens move to preferential gathering sites is decreasing with the increase of the lattice size.

We also consider the issue of finding the time when the situation on the lattice is stabilized, i.e. all citizens who can go active are positioned around preferential gathering sites. We consider the case when active citizens move to the preferential gathering site with 100% probability. We use the optimal number of LEOs per 1,000 citizens found earlier so that the number of active citizens per day is 0.2%. We consider three lattices sizes (100x100, 200x200, 300x300). 100,101 days on the lattice were computed for each simulation. Each experiment was repeated twice. We found out that in the case of a small lattice, a large amount of time around 60000-90000 days is required till the situation is stabilized. For the lattice of size 200x200 around 30000 days was enough. For the lattice of size 300x300 the situation was stabilized after about 60000 days.

4 Effect of Remote Vision via A Small World Network

In this Section we introduce a small world network into our agent-based model. About 90% of neighbors of a citizen remain in his/her local vision, while 10% of neighbors (distant neighbors) are visible remotely through the small world network.
The small world network is constructed as an undirected graph with N nodes and $N * K/2$ edges, where the number of nodes N is the population size and the mean degree K is the number of distant neighbors for every citizen. First a regular ring lattice is constructed, which is a graph with N nodes each connected to K neighbors, $K/2$ on each side. Then for every node $n_i = n_0, \dots, n_{N-1}$ we take every edge (n_i, n_j) with $i < j$, and rewire it with probability β ($0 \leq \beta \leq 1$). Rewiring is done by replacing (n_i, n_j) with (n_i, n_k) where k is chosen with uniform probability from all possible values that avoid loops ($k \neq i$) and link duplication (there is no edge $(n_i, n_{k'})$ with $k' = k$ at this point in the algorithm).
We consider the following model outcomes: number of active citizens per day in percent, number of revolutions per 1,000 days, peak number of active citizens in percent, number of jailed citizens before outburst is just about to start, number of violent outbursts per year, rate of violence. We introduce a threshold of 5% of population and call an outburst large-scale if it exceeds the threshold. The number of active citizens per day is the total number of citizens active at the end of the day. The number of revolutions per 1,000 days is the total number of large-scale outbursts per 1,000 days. The peak number of active citizens is the maximum number of active citizens during the outburst. The number of jailed citizens before outburst is just about to start is the minimum number of jailed citizens within 20 days before the peak of the outburst. The number of violent outbursts per year is inversely proportional to the waiting time between two large-scale outbursts. The rate of violence is the number of citizens active at the end of the day or jailed during the day per 1,000 citizens.

4.1 Test Case with Small Number of LEOs per 1,000 Citizens

We consider that citizens density is 0.7 (i.e. citizens occupy 70% of the lattice); number of LEOs per 1,000 citizens is 1.59; LEO vision is 10.5; LEO speed is 4 (i.e. LEOs move 4 times per day); maximum jail term is 120 days; fraction of always active citizens R = 0.025; fraction of never active citizens G = 0.5; fraction of conditionally active citizens is 0.475. By using the same vision for citizens as the vision for LEOs, we calculate that the initial number of neighbors for a citizen is 244.3, which is the total number of cells in their vision multiplied by the population density. The number of distant neighbors is about 10% of the total number of neighbors. Varying citizen vision, we get that the citizen vision of 10.01 gives 221.9 neighbors in vision and we introduce 22 distant neighbors in the small world network. Threshold T and legitimacy L can be calculated by using formulas (2) and (3). All always active citizens are initially in jail. 1,000 days on the lattice were computed for each simulation. We consider lattices of sizes 100x100, 200x200, 300x300 for the case without small world network and the case with small world network and $\beta = 0.2$ and $\beta = 0.8$. Each experiment was repeated at least 10 times. For testing purposes the number of LEOs per 1,000 citizens is selected in such a way so that the rate of violence is high and does not depend on the lattice size but the jail term and LEO speed.
Figures 26 and 27 show positions of citizens and LEOs just before the outburst begins and during the outburst for the lattice of size 200x200 in the cases without the small world network and with the small world network and $\beta = 0.2$. Figure 28 shows the number of actives and jailed for the lattice of size 200x200 in the cases without the small world network and with the small world network and $\beta = 0.2$. From the graphical representation we can see that in the case with the small world network there are more citizens on the lattice and there is fewer activity between outbursts.

[Figure 26 here.]

[Figure 27 here.]

[Figure 28 here.]

The results for lattices of sizes 100x100, 200x200, 300x300 for the case without small world network and the case with small world network and $\beta = 0.2$ and $\beta = 0.8$ are plotted in Figures 29, 30, 31, 33, 32, 34. The cases with different β produce similar results, because the small world network is built on already randomized positions of citizens.

[Figure 29 here.]

[Figure 30 here.]

[Figure 31 here.]

[Figure 32 here.]

[Figure 33 here.]

[Figure 34 here.]

The number of active citizens per day and rate of violence are comparable for the cases without small world network and with small world network. This can be explained by the fact that the number of LEOs per 1,000 citizens is small and the total number of actives per day is saturated by the jail term and the total number of actives LEOs can arrest per day. The peak number of active citizens increases substantially after introducing small world network. In the case with small world network there is fewer activity between outbursts, there are less jailed citizens and there are more citizens on the lattice capable of going active during the outburst. This can be explained by citizen's local vision being smaller than in the case without small world network. The number of revolutions and the number of violent outbursts per year increase substantially after introducing small world network. In the case with small world network peak numbers of active citizens are bigger and more outbursts of activity go over 5% threshold to be considered and counted as large-scale outbursts.

4.2 Case with the Realistic Number of LEOs per 1,000 Citizens

We consider that citizens density is 0.7; LEO vision is 14; LEO speed is 4; maximum jail term is 120 days; fraction of always active citizens R = 0.025; fraction of never active citizens G = 0.5; fraction of conditionally active citizens is 0.475. By using the same vision for citizens as the vision for LEOs, we calculate that the initial number of neighbors for a citizen is 429.1. The number of distant neighbors is about 10% of the total number of neighbors. Varying citizen vision, we get that the citizen vision of 13.16 gives 387.1 neighbors in vision and we introduce 42 distant neighbors in the small world network. We also consider the case with introducing the small world network with about 10% of the total number of neighbors, which is 42 in our case, but leaving the citizen vision as 14. Threshold T and legitimacy L can be calculated by using formulas (2) and (3). All always active citizens are initially in jail. 50,000 days on the lattice were computed for each simulation. We consider lattices of sizes 100x100, 200x200, 300x300 for the case without small world network and the case with small world network. The cases with different β produce similar results, so we only consider the case of $\beta = 0.8$. Number of LEOs per 1,000 citizens is 1.51 for lattice 100x100, number of LEOs per 1,000 citizens is 1.77 for lattice 200x200, number of LEOs per 1,000 citizens is 1.87 for lattice 300x300. Each experiment was repeated at least 28 times.
Figure 35 shows the number of actives and jailed for the lattice of size 200x200 in the cases without the small world network and with the small world network and citizen vision of 13.16 and citizen vision of 14. From the graphical representation we can see that in the case with the small world network and smaller citizen vision there are much fewer outbursts. This suggests that the local vision is important.

[Figure 35 here.]

The results for lattices of sizes 100x100, 200x200, 300x300 for the case without small world network and the case with small world network and citizen vision of 13.16 and citizen vision of 14 are plotted in Figures 36, 37, 38, 40, 39, 41.

[Figure 36 here.]

[Figure 37 here.]

[Figure 38 here.]

[Figure 39 here.]

[Figure 40 here.]

[Figure 41 here.]

The number of active citizens per day and rate of violence are much lower for the case with small world network and smaller citizen vision. The peak number of active citizens is the lowest for the case without small world network and then increases a little with introducing small world network, although the peak number of active citizens is slightly higher for the case with smaller citizen vision. In the case with smaller citizen vision there are less jailed citizens before the outburst is about to begin and there are more citizens on the lattice capable of going active during the outburst. The number of revolutions and the number of violent outbursts per year are much lower for the case with small world network and smaller citizen vision. In conclusion, when an adequate number of law enforcement offices is maintained, citizens' local vision is the main factor determining the rate of violence / criminal activity. Unlike the case of low LEO density, in the case of adequate LEO density replacing a fraction of neighbors in citizens' local vision with the same number of remote neighbors doesn't make a city more violent / criminal.

References

[1] Epstein JM. Modeling civil violence; an agent-based computational approach. Proc. Natl. Acad. Sci. USA. 2002; 99:7243–7250. 3

[2] Fonoberova M, Fonoberov VA, Mezic I, Mezic J, Brantingham J. Nonlinear Dynamics of Crime and Violence in Urban Settings. (submitted to Journal of Artificial Societies and Social Simulation). 3, 5

[3] Fonoberova M, Fonoberov VA, Mezic I. Global Sensitivity/Uncertainty Analysis for Agent-Based Models. (submitted to Reliability Engineering and System Safety). 5

[4] Zou Y, Fonoberov V, Fonoberova M, Mezic I, Kevrekidis I. A Case Study of Model Reduction for Agent-Based Social Simulation: Coarse-Graining a Civil Violence Model. (accepted for publication in Physical Review E). 5

[5] Global Optimization, Sensitivity and Uncertainty in Models (GoSUM) Software, Aimdyn, Inc. Available at http://aimdyn.com/software/

Figure 1: Sketch of the model. Currently quiescent (active) citizens are represented by blue (red) cells, LEOs are represented by black cells. Yellow cells show Moore neighborhood for an agent. During a move, an agent picks a random cell in his/her neighborhood and if the selected cell is unoccupied, he/she moves there, otherwise the agent stays put. Pink cells show a vision of radius 4 for the LEO (black cell). The LEO arrests the nearest active citizen its field of vision. Green cells show vision of radius 4 for the citizen (blue cell). The citizens calculate their state depending on conditions on the lattice within their field of vision.

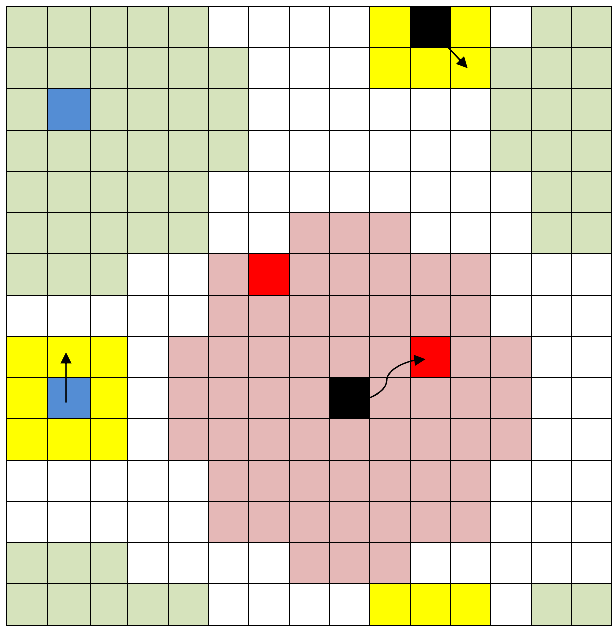

Figure 2: Citizen's perceived risk function.

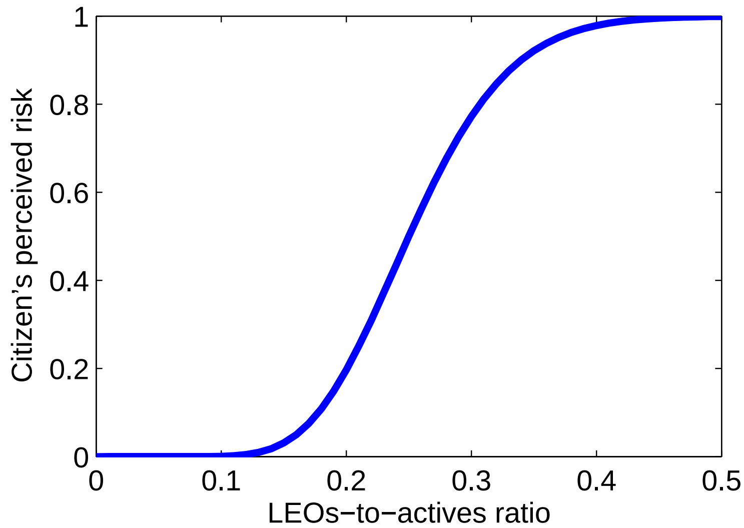

Figure 3: Citizen state as a function of threshold. Green - fraction of citizens who are never active, red - fraction of citizens who are always active, yellow - fraction of citizens who use arrest probability to decide their state.

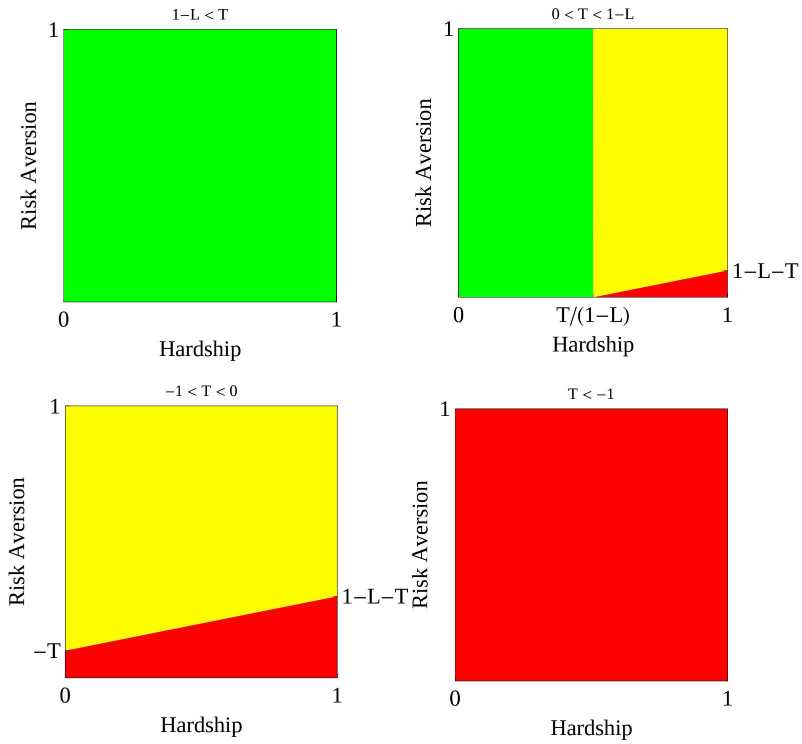

Figure 4: The active citizen (A) can retain its active state, go to jail (J) if the LEO arrests him/her, or become quiescent (Q) if the difference between the citizen's grievance E and the citizen's perceived net risk N is smaller than the specified threshold. The quiescent citizen (Q) can retain its quiescent state or become active (A) if the difference between the citizen's grievance E and the citizen's perceived net risk N is bigger or equal to the specified threshold. The jailed citizen (J) becomes quiescent (Q) when its jail term (JT) is over.

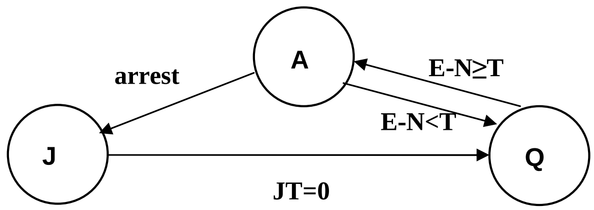

Figure 5: Actives (red) and jailed (black) citizens (in percent) for the case of no preferential gathering sites.

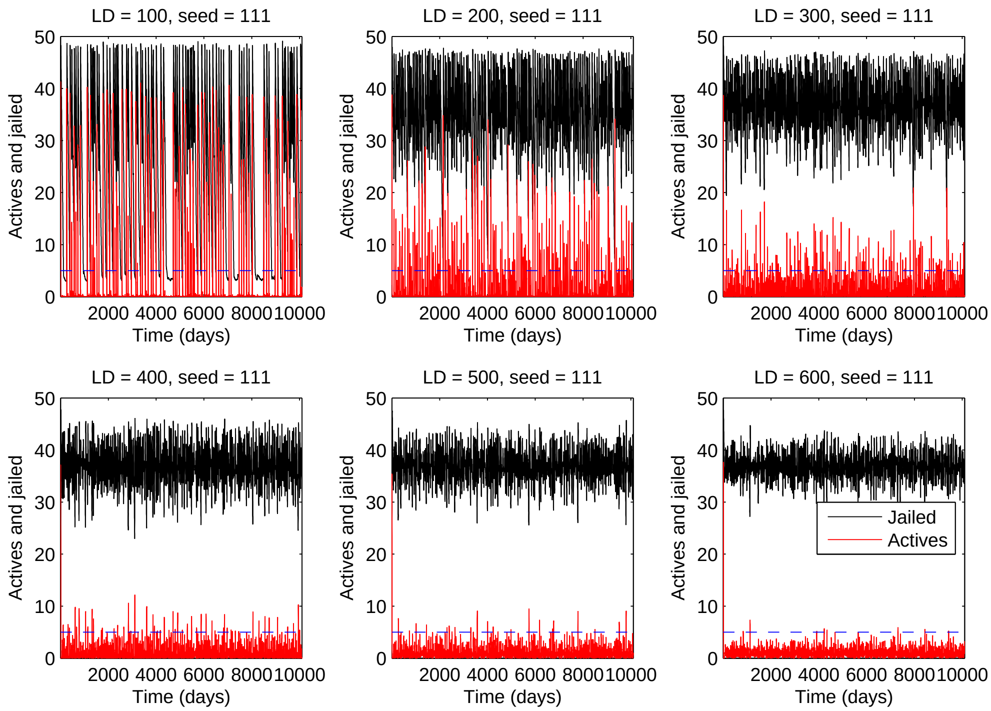

Figure 6: Actives (red) and jailed (black) citizens (in percent) for the case of 3 preferential gathering sites.

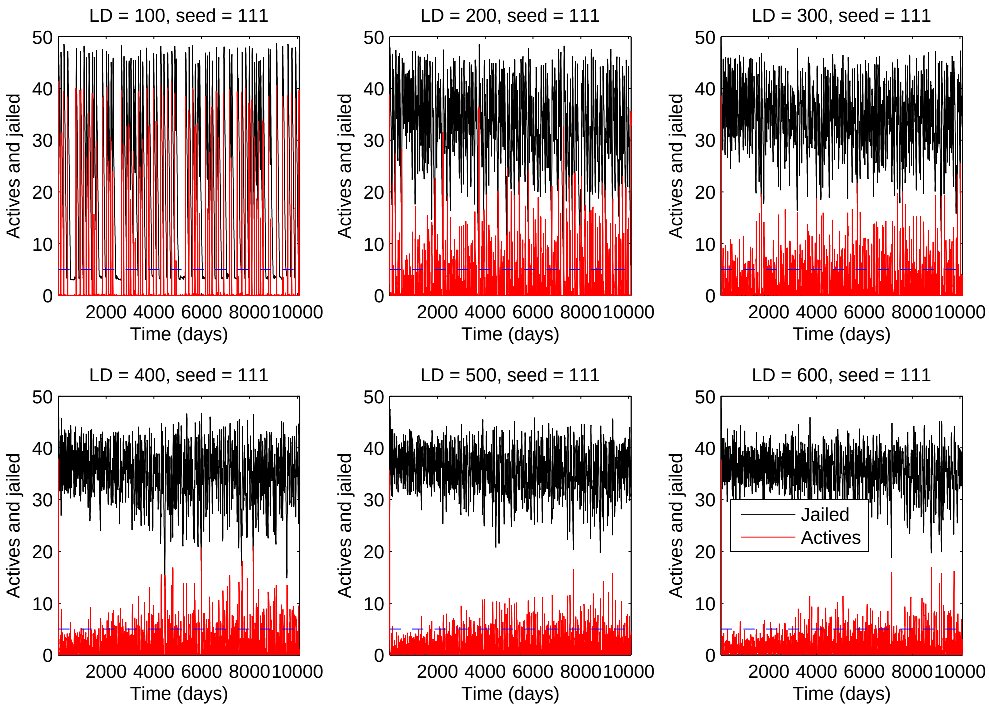

Figure 7: Lattice situation at day 10040 and day 10070. Citizens, who can become active, are colored blue if quiescent and red if active. LEOs are colored black, never active citizens and unoccupied sites are white.

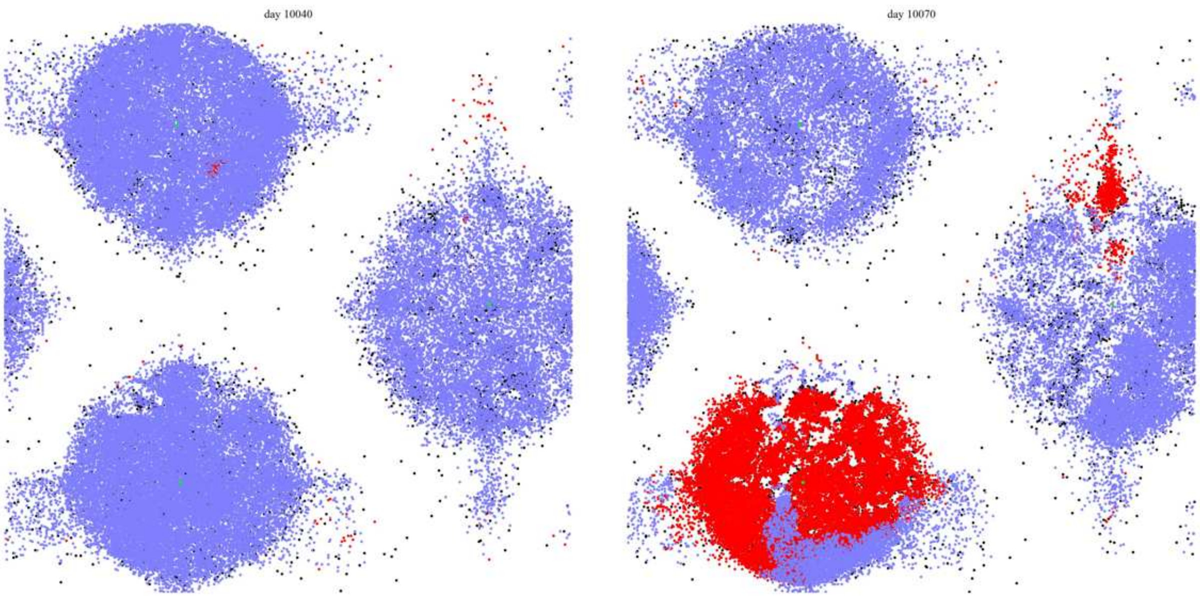

Figure 8: Actives (red) and jailed (black) citizens for the case of 30 preferential gathering sites.

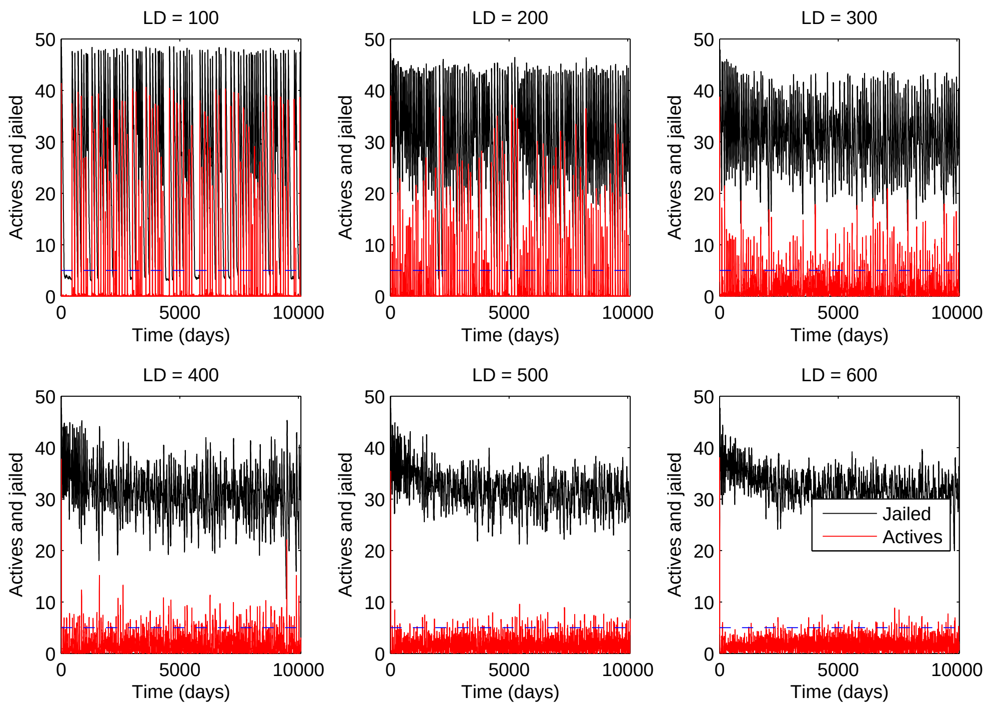

Figure 9: Number of active citizens per day (in percent) for different numbers of preferential gathering sites and lattice sizes of 100x100, 200x200, 300x300, 400x400, 500x500, 600x600.

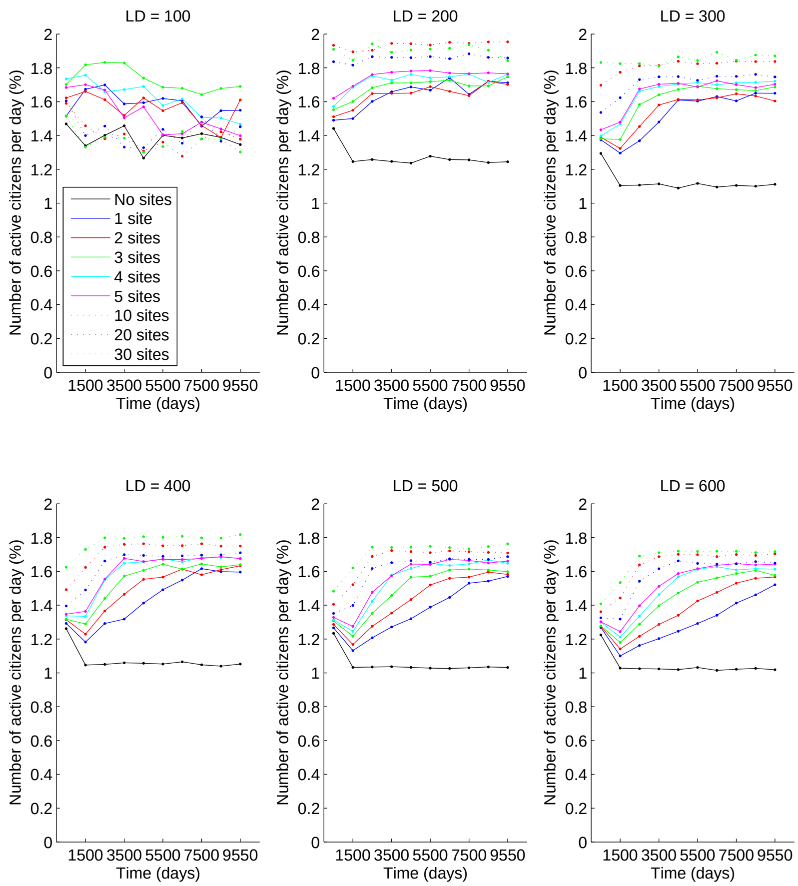

Figure 10: Number of revolution per 1,000 days for different numbers of preferential gathering sites and lattice sizes of 100x100, 200x200, 300x300, 400x400, 500x500, 600x600.

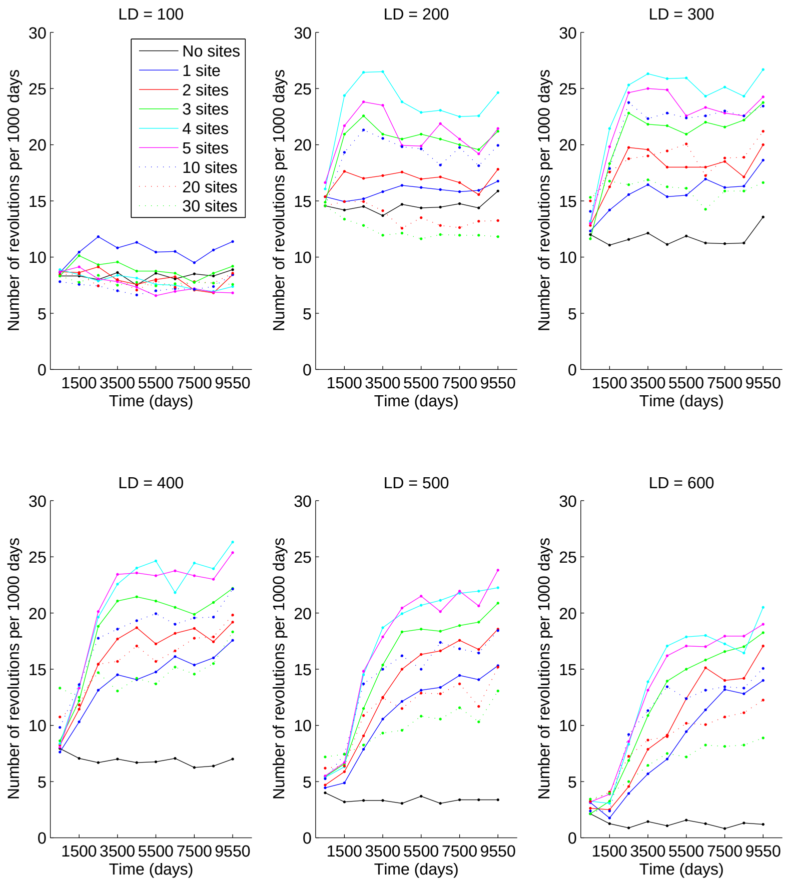

Figure 11: Peak number of active citizens (in percent) for different numbers of preferential gathering sites and lattice sizes of 100x100, 200x200, 300x300, 400x400, 500x500, 600x600.

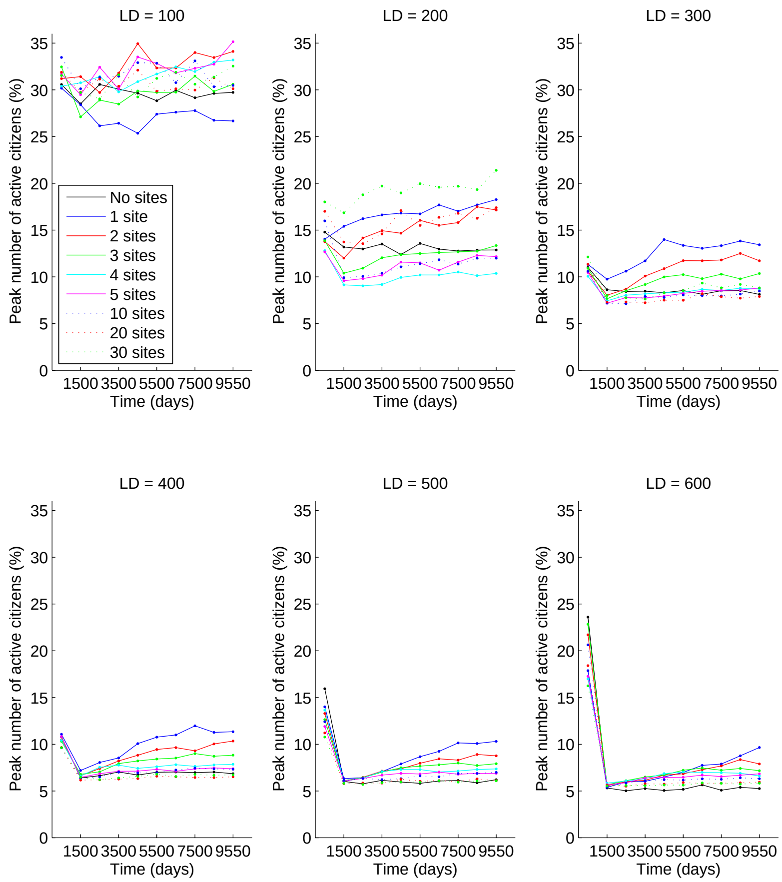

Figure 12: Number of violent outbursts per year for different numbers of preferential gathering sites and lattice sizes of 100x100, 200x200, 300x300, 400x400, 500x500, 600x600.

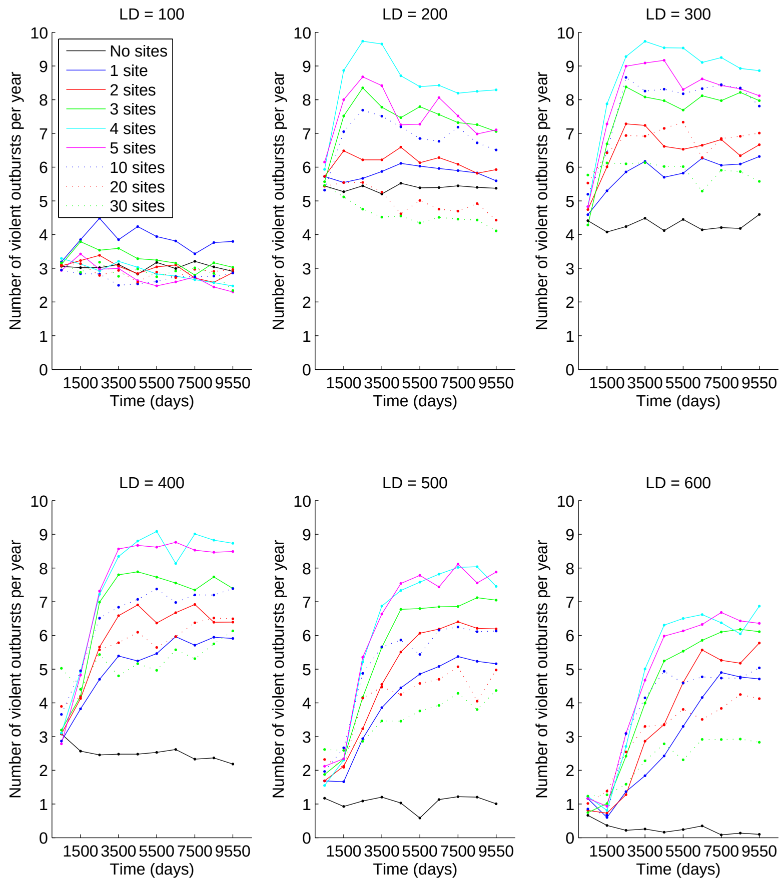

Figure 13: Rate of violence for different numbers of preferential gathering sites and lattice sizes of 100x100, 200x200, 300x300, 400x400, 500x500, 600x600.

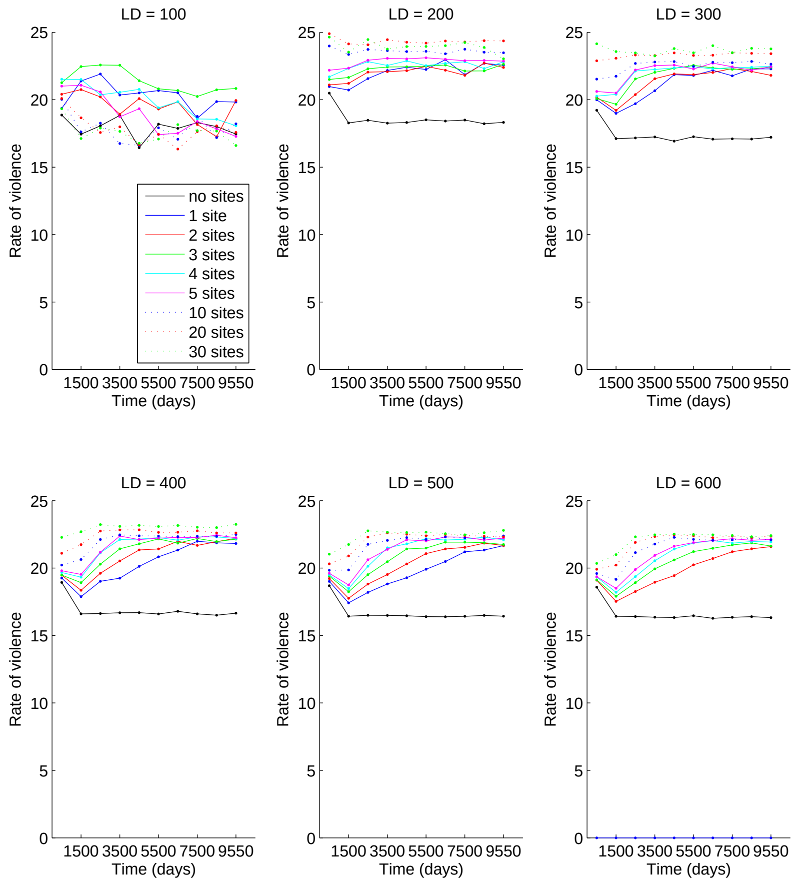

Figure 14: Number of active citizens per day (in percent) for different numbers of sites in dependence on lattice size.

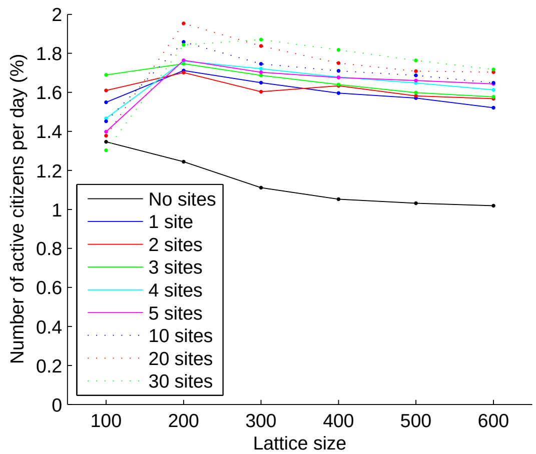

Figure 15: Number of revolution per 1,000 days and peak number of active citizens (in percent) for different numbers of sites in dependence on lattice size.

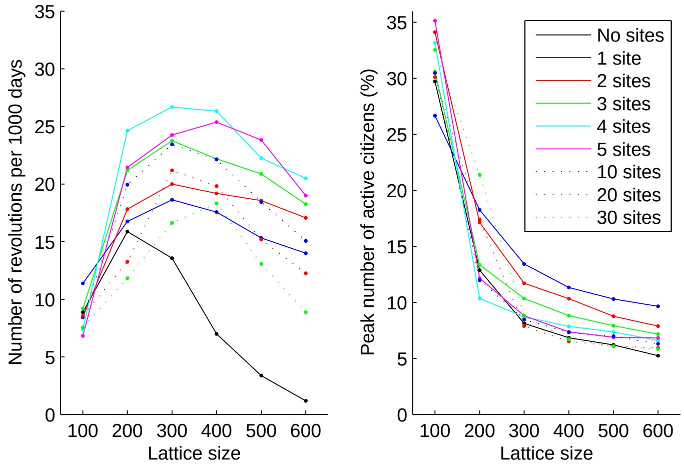

Figure 16: Number of violent outbursts per year and rate of violence for different numbers of sites in dependence on lattice size.

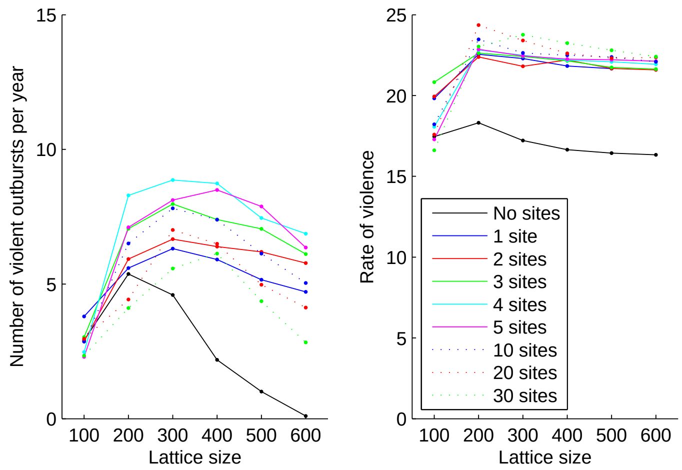

Figure 17: Number of active citizens per day (in percent) for five preferential gathering sites in dependence on lattice size and probability with which active citizens move to preferential gathering sites.

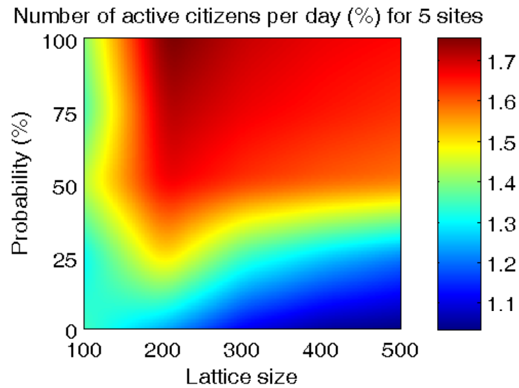

Figure 18: Number of revolutions per 1,000 days and peak number of active citizens (in percent) for five preferential gathering sites in dependence on lattice size and probability with which active citizens move to preferential gathering sites.

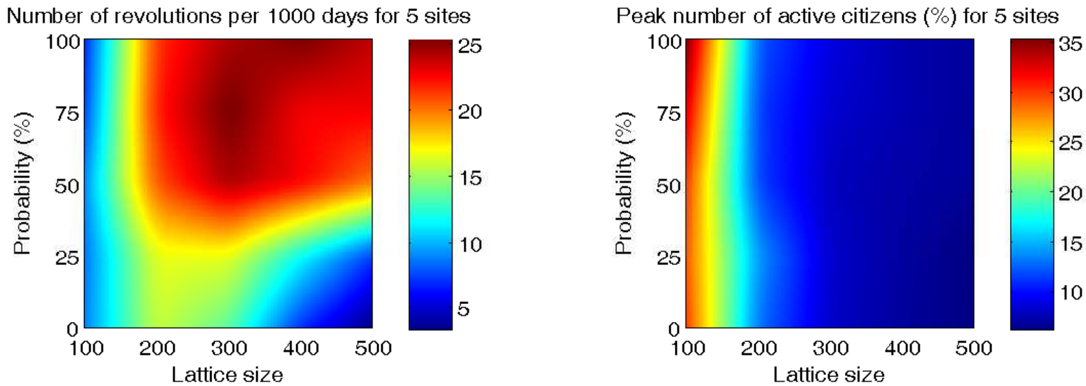

Figure 19: Number of violent outbursts per year and rate of violence for five preferential gathering sites in dependence on lattice size and probability with which active citizens move to preferential gathering sites.

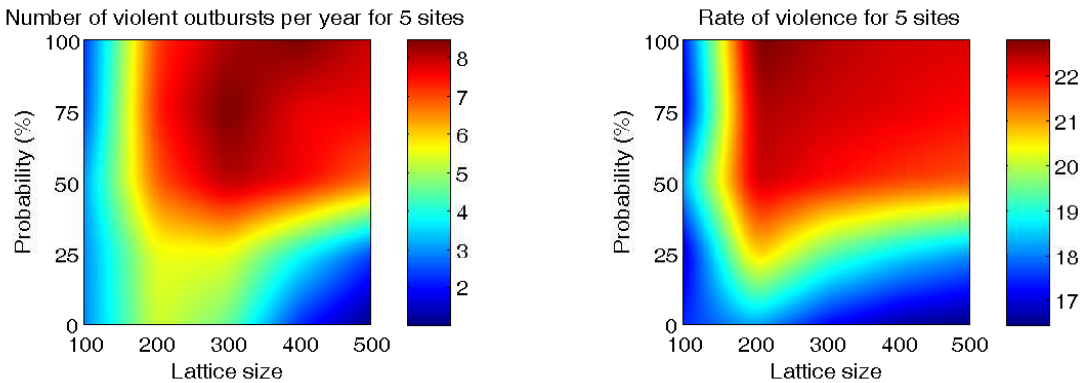

Figure 20: Number of active citizens per day in dependence on number of LEOs per 1,000 citizens for five preferential gathering sites and lattice sizes of 100x100, 200x200, 300x300.

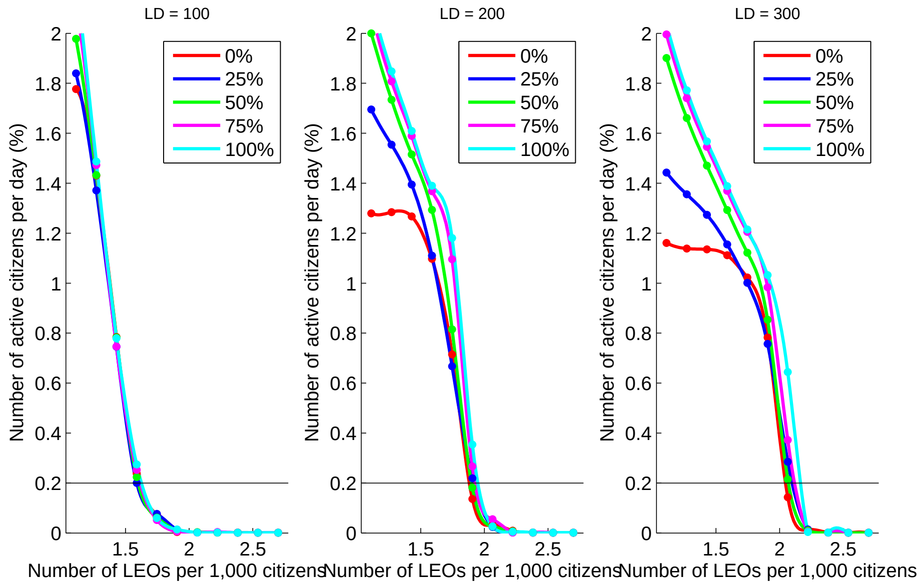

Figure 21: Number of active citizens per day (in percent) for different probabilities in dependence on lattice size and for different lattice sizes in dependence on probability.

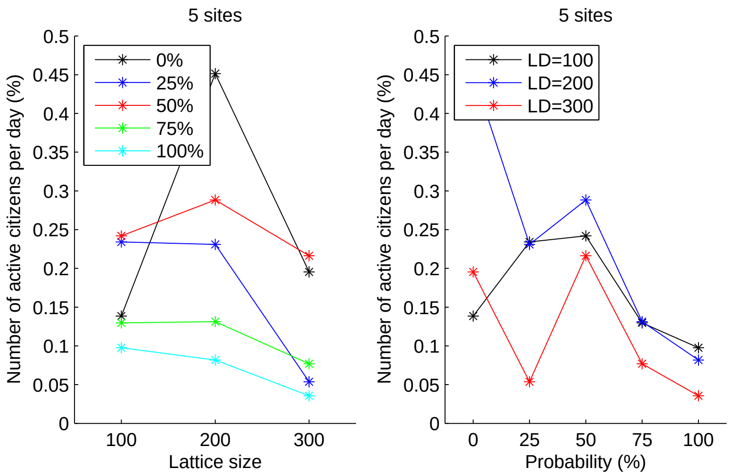

Figure 22: Number of revolution per 1,000 days for different probabilities in dependence on lattice size and for different lattice sizes in dependence on probability.

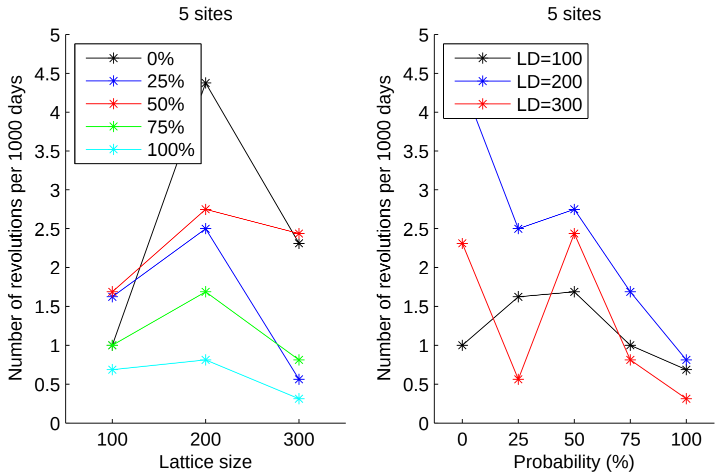

Figure 23: Peak number of active citizens (in percent) for different probabilities in dependence on lattice size and for different lattice sizes in dependence on probability.

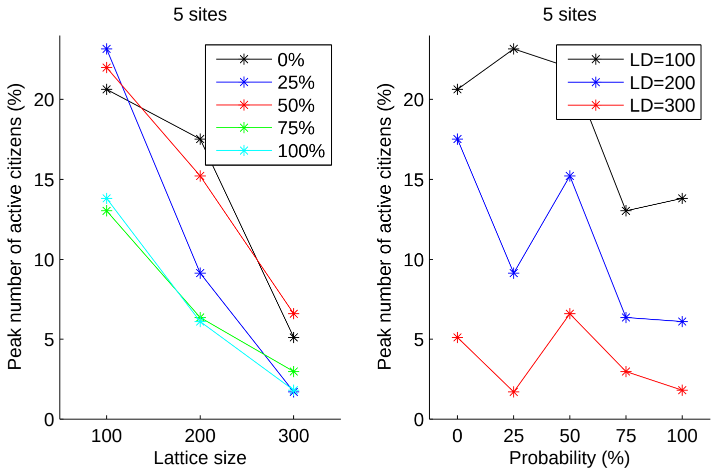

Figure 24: Number of violent outbursts per year for different probabilities in dependence on lattice size and for different lattice sizes in dependence on probability.

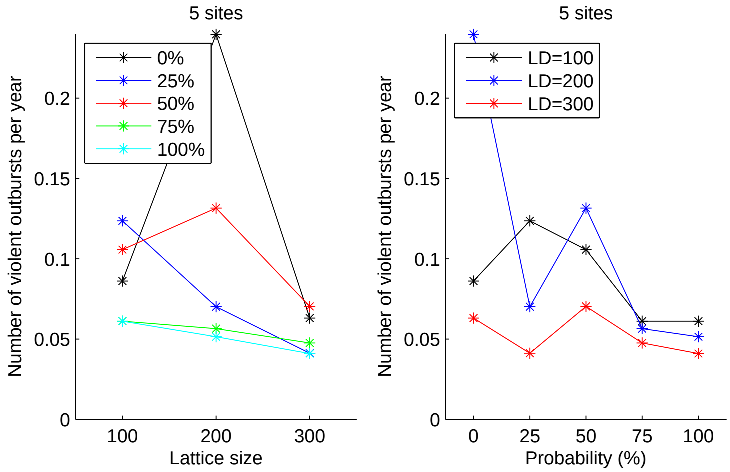

Figure 25: Rate of violence for different probabilities in dependence on lattice size and for different lattice sizes in dependence on probability.

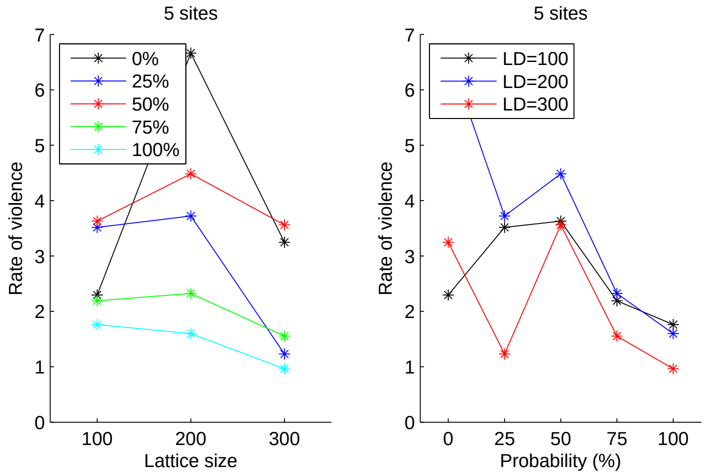

Figure 26: Lattice situation at day 981 and day 989 for the lattice of size 200x200 in the case without the small world network. Citizens, who can become active, are colored blue if quiescent and red if active. LEOs are colored black, never active citizens and unoccupied sites are white.

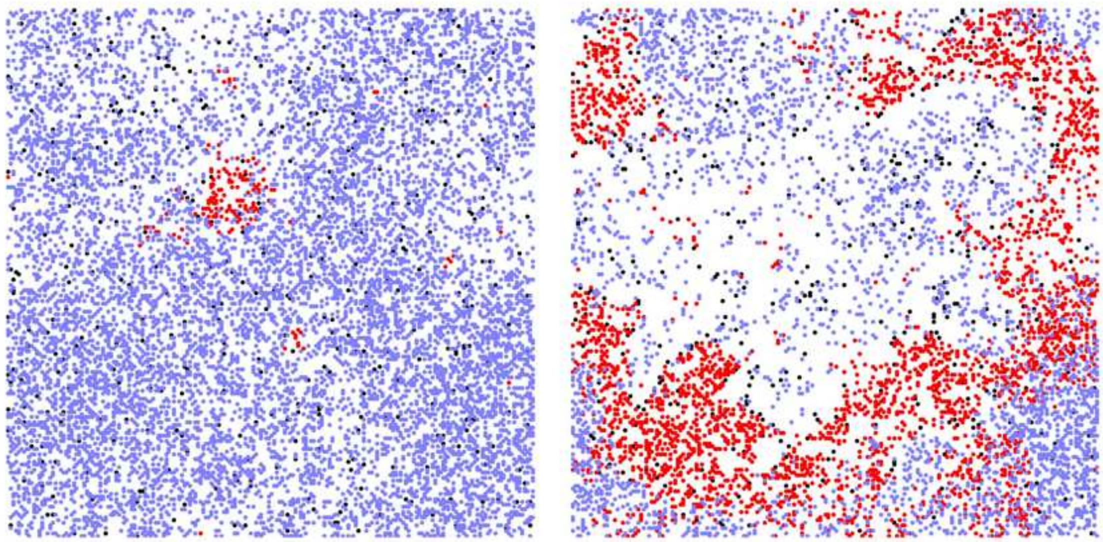

Figure 27: Lattice situation at day 214 and day 220 for the lattice of size 200x200 in the case with the small world network. Citizens, who can become active, are colored blue if quiescent and red if active. LEOs are colored black, never active citizens and unoccupied sites are white.

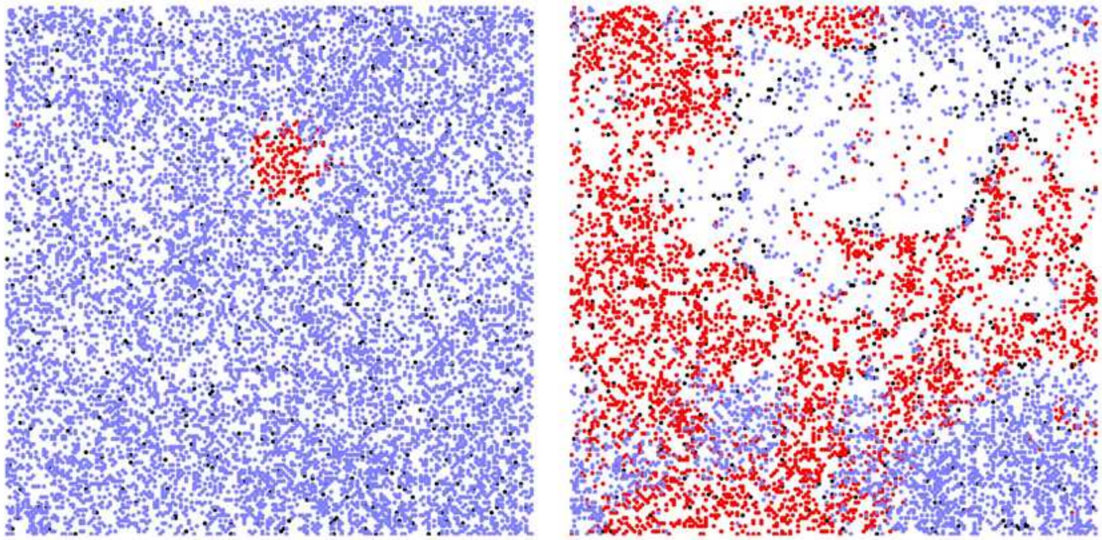

Figure 28: Number of active and jailed citizens in percent for the lattice of size 200x200 in the cases without the small world network and with the small world network and $\beta = 0.2$.

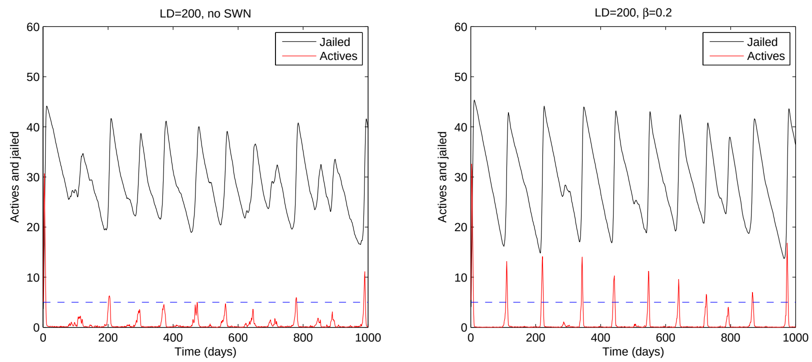

Figure 29: Number of active citizens per day in percent. In the case without small world network citizen vision is 10.5, number of cells in vision is 349, average number of neighbors in vision is 244.3. In the case with small world network citizen vision is 10.01, number of cells in vision is 317, average number of neighbors in vision is 221.9, number of citizens in vision through small world network is 22.

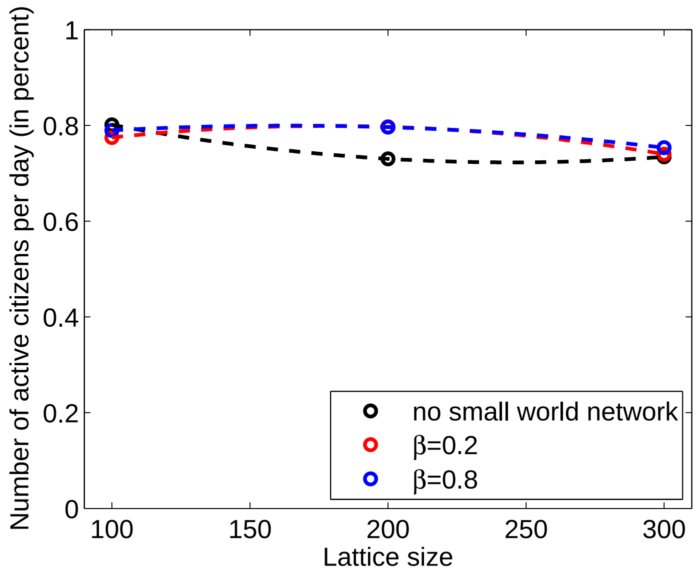

Figure 30: Number of revolutions. In the case without small world network citizen vision is 10.5, number of cells in vision is 349, average number of neighbors in vision is 244.3. In the case with small world network citizen vision is 10.01, number of cells in vision is 317, average number of neighbors in vision is 221.9, number of citizens in vision through small world network is 22.

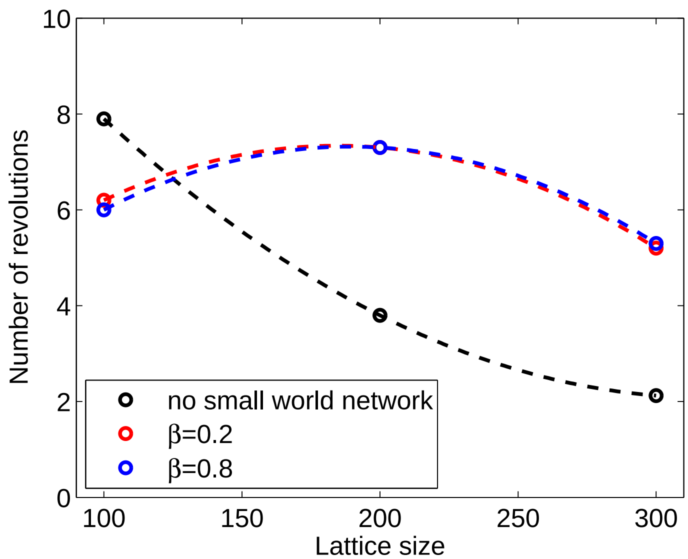

Figure 31: Peak number of active citizens in percent. In the case without small world network citizen vision is 10.5, number of cells in vision is 349, average number of neighbors in vision is 244.3. In the case with small world network citizen vision is 10.01, number of cells in vision is 317, average number of neighbors in vision is 221.9, number of citizens in vision through small world network is 22.

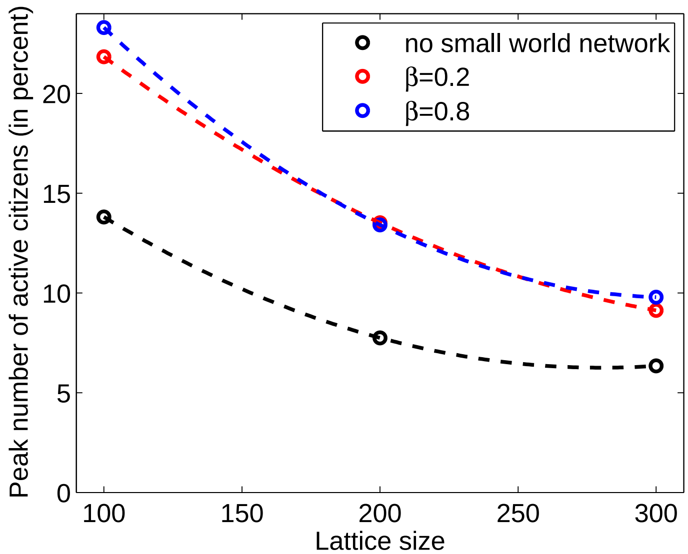

Figure 32: Number of jailed citizens before outburst is just about to start in percent. In the case without small world network citizen vision is 10.5, number of cells in vision s 349, average number of neighbors in vision is 244.3. In the case with small world network citizen vision is 10.01, number of cells in vision is 317, average number of neighbors in vision is 221.9, number of citizens in vision through small world network is 22.

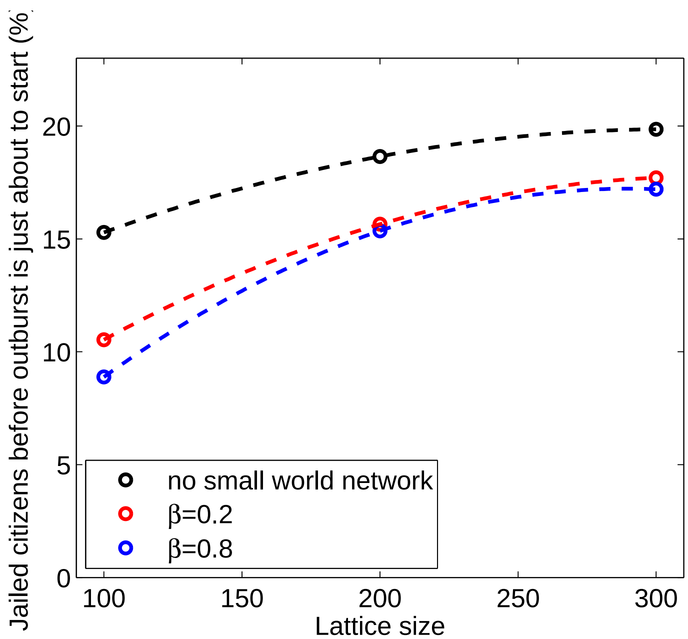

Figure 33: Number of violent outbursts per year. In the case without small world network citizen vision is 10.5, number of cells in vision is 349, average number of neighbors in vision is 244.3. In the case with small world network citizen vision is 10.01, number of cells in vision is 317, average number of neighbors in vision is 221.9, number of citizens in vision through small world network is 22.

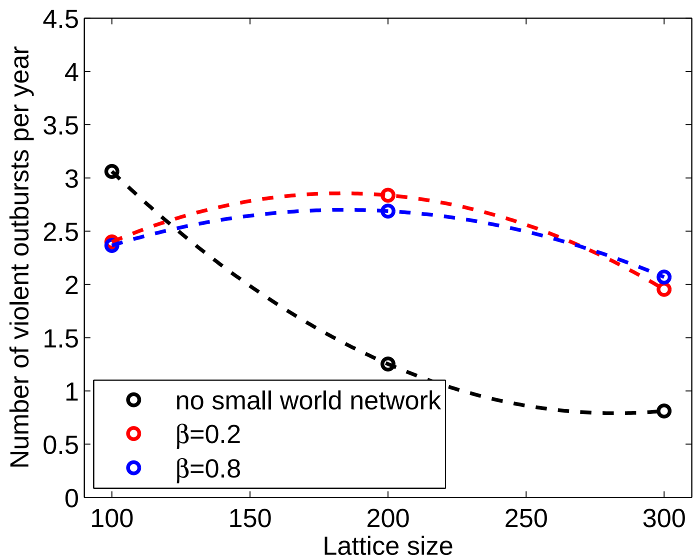

Figure 34: Rate of violence. In the case without small world network citizen vision is 10.5, number of cells in vision is 349, average number of neighbors in vision is 244.3. In the case with small world network citizen vision is 10.01, number of cells in vision is 317, average number of neighbors in vision is 221.9, number of citizens in vision through small world network is 22.

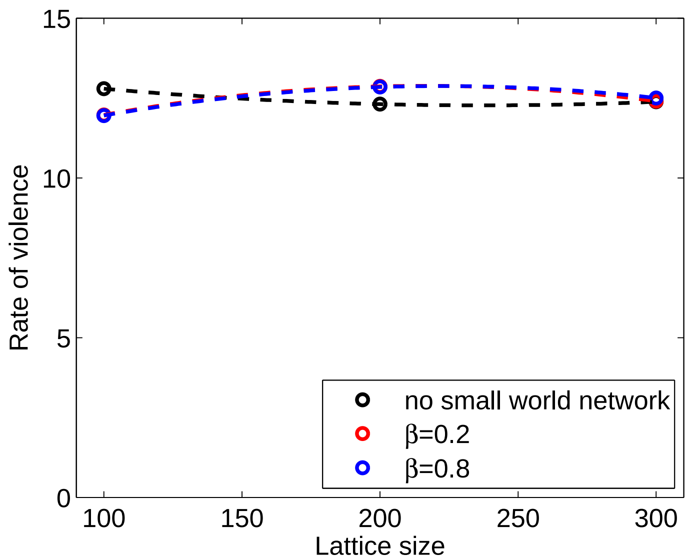

Figure 35: Number of active and jailed citizens in percent for the lattice of size 200x200 in the realistic cases without the small world network and with the small world network and $\beta = 0.2$.

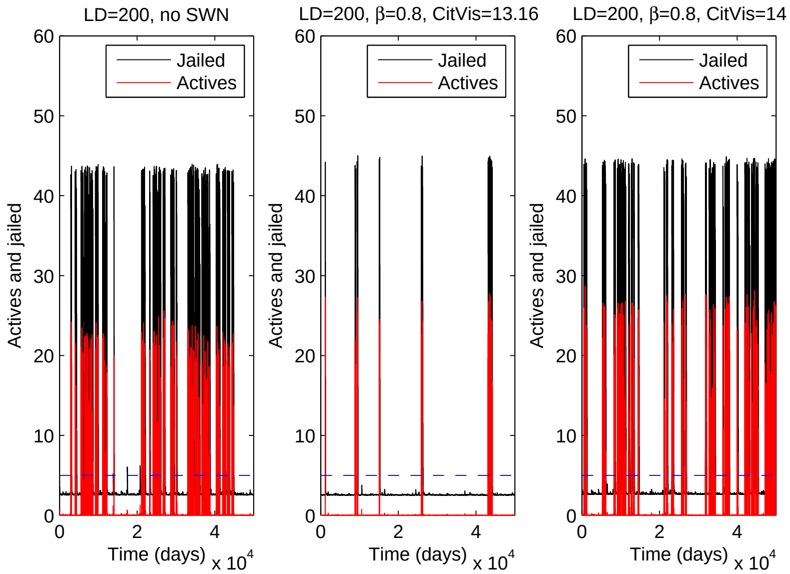

Figure 36: Number of active citizens per day in percent for realistic cases. In the case without small world network citizen vision is 14, number of cells in vision is 613, average number of neighbors in vision is 429.1. In the first case with small world network citizen vision is 13.16, number of cells in vision is 553, average number of neighbors in vision is 387.1, number of citizens in vision through small world network is 42. In the second case with small world network citizen vision is 14, number of cells in vision is 613, average number of neighbors in vision is 429.1, number of citizens in vision through small world network is 42.

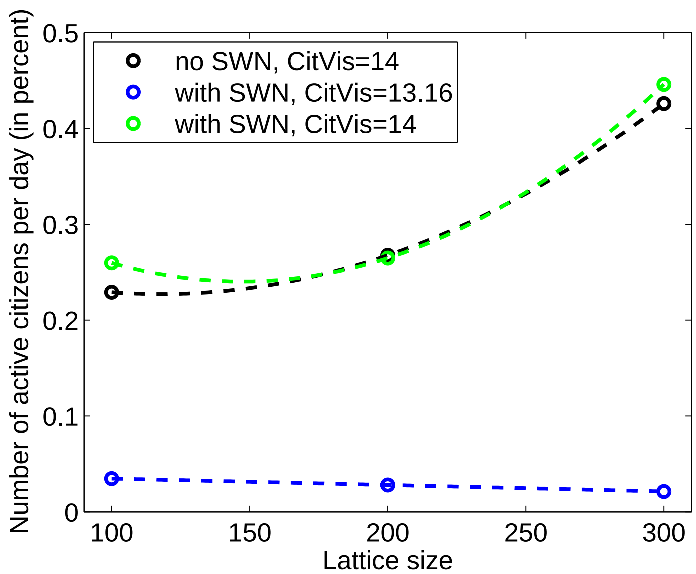

Figure 37: Number of revolutions for realistic cases. In the case without small world network citizen vision is 14, number of cells in vision is 613, average number of neighbors in vision is 429.1. In the first case with small world network citizen vision is 13.16, number of cells in vision is 553, average number of neighbors in vision is 387.1, number of citizens in vision through small world network is 42. In the second case with small world network citizen vision is 14, number of cells in vision is 613, average number of neighbors in vision is 429.1, number of citizens in vision through small world network is 42.

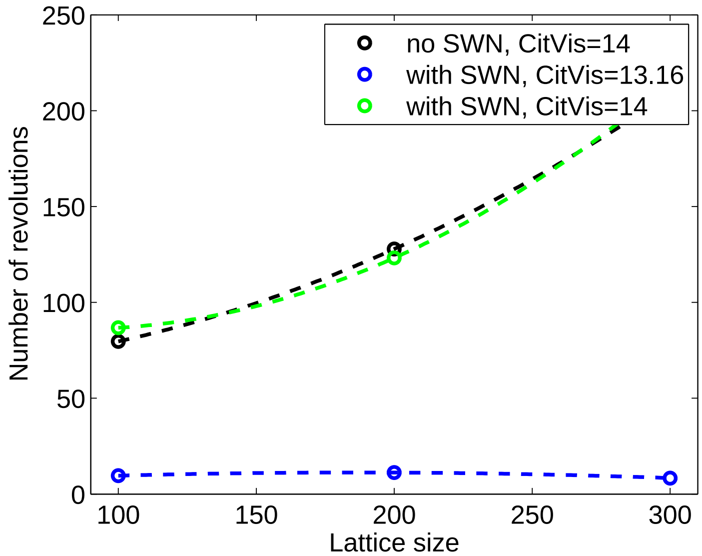

Figure 38: Peak number of active citizens in percent for realistic cases. In the case without small world network citizen vision is 14, number of cells in vision is 613, average number of neighbors in vision is 429.1. In the first case with small world network citizen vision is 13.16, number of cells in vision is 553, average number of neighbors in vision is 387.1, number of citizens in vision through small world network is 42. In the second case with small world network citizen vision is 14, number of cells in vision is 613, average number of neighbors in vision is 429.1, number of citizens in vision through small world network is 42.

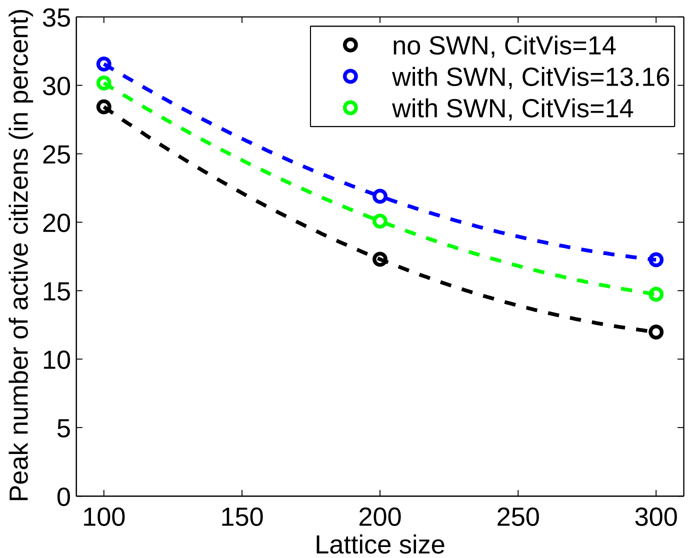

Figure 39: Number of jailed citizens before outburst is just about to start in percent for realistic cases. In the case without small world network citizen vision is 14, number of cells in vision is 613, average number of neighbors in vision is 429.1. In the first case with small world network citizen vision is 13.16, number of cells in vision is 553, average number of neighbors in vision is 387.1, number of citizens in vision through small world network is 42. In the second case with small world network citizen vision is 14, number of cells in vision is 613, average number of neighbors in vision is 429.1, number of citizens in vision through small world network is 42.

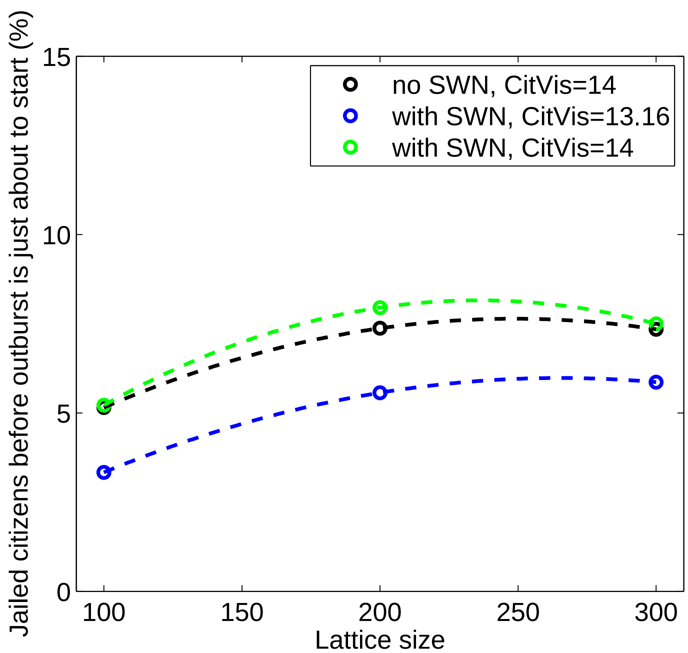

Figure 40: Number of violent outbursts per year for realistic cases. In the case without small world network citizen vision is 14, number of cells in vision is 613, average number of neighbors in vision is 429.1. In the first case with small world network citizen vision is 13.16, number of cells in vision is 553, average number of neighbors in vision is 387.1, number of citizens in vision through small world network is 42. In the second case with small world network citizen vision is 14, number of cells in vision is 613, average number of neighbors in vision is 429.1, number of citizens in vision through small world network is 42.

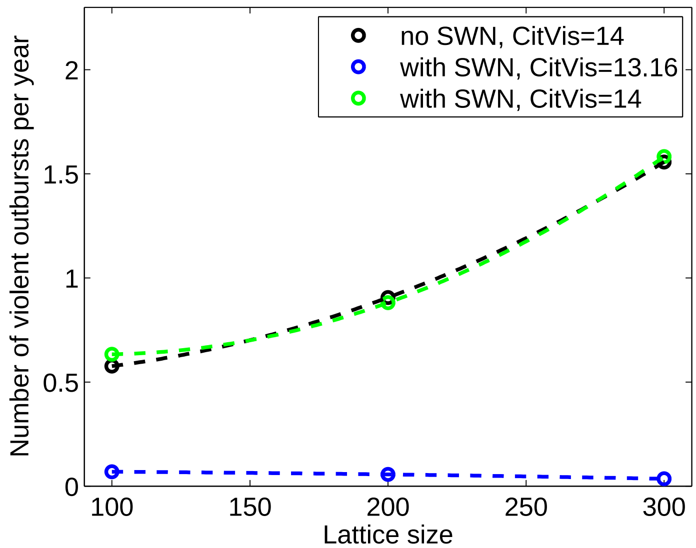

Figure 41: Rate of violence for realistic cases. In the case without small world network citizen vision is 14, number of cells in vision is 613, average number of neighbors in vision is 429.1. In the first case with small world network citizen vision is 13.16, number of cells in vision is 553, average number of neighbors in vision is 387.1, number of citizens in vision through small world network is 42. In the second case with small world network citizen vision is 14, number of cells in vision is 613, average number of neighbors in vision is 429.1, number of citizens in vision through small world network is 42.